A Delay-Reliability Analysis for Multihop Underwater Acoustic Communication

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ABSTRACT

This paper investigates the delay-reliability tradeoff for multihop underwater acoustic networks. The propagation medium of underwater acoustic channel exhibits distinct characteristics when contrasted with other common propagation media such as copper, fiber, and radio. In particular there are the extremely slow propagation speed of sound in water, high signal attenuation due to absorption, significant delay spreads and intersymbol interference, and range-dependent transmission bandwidth. These features make the delay-reliability tradeoff for underwater acoustic channels fundamentally different from other channels. The approach is based on error-exponents which enable a physical-layer comparison of multihopping versus no hops while considering the overall throughput. The analysis shows that for typical network parameters, increasing the number of hops dramatically improves both the achievable information rate and the achievable reliability function, which quantitatively captures the decay rate of the decoding error probability as the coding block length increases asymptotically. Numerical results are presented to illustrate the analysis.

Categories and Subject Descriptors

H.1.1 [**Information Systems**]: Systems and information theory

General Terms

Theory

Keywords

Underwater acoustic communications, underwater acoustic networks, multihop networks.

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1. INTRODUCTION

Multihop networks, composed of a cascade of point-to-point channels, or, hops, have been persistently studied in various settings; see [3]–[9]. Rather generally, the main benefit of multihopping stems from a simple fact: "two hops over strong links are better than one hop over a weak link." [10]. Additionally, in a wireless environment, multiple relay nodes along with the source can (at least in principle) transmit cooperatively such that a certain degree of spatial diversity may be achieved; see, *e.g.*, [11][12]. Thus, multihopped wireless transmission has emerged as a method by which to improve resource use in cellular and *ad hoc* networks. Due to the severe underwater channel characteristics, multihopped networks offer even more promise for increasing throughput and performance [2, 1]. In [1], we examined the potential gains of multihopping and cooperative communications in an underwater setting. Through an end-to-end probability of bit error analysis, we showed that the gains due to multihopping and cooperation could be significant (up to orders of magnitude). However, the analysis of [1] was based on the transmission of bits and ignored the half duplex nature of acoustic modems. These two issues are inter-related; they imply that we cannot examine the transmission of isolated bits, but rather need to consider the overall rate of transmission; to this end, delay will play a major role. Thus, we turn to information theoretic analyses which enable the consideration of performance (reliability) as well as rate.

Thus, we need to consider several important aspects of practical transmission especially in the context of systems not built upon wired media such as copper or fiber. Key to this is the consideration of half duplex transceivers. Due to inter-hop interference and the half-duplex constraint, relay nodes need to adopt certain spatial link access modes such as orthogonal time division or periodic spatial reuse. A downward scaling effect in the end-to-end spectral efficiency can emerge as the number of hops increases. This observation leads to the recognition that there usually exists an optimal number of hops, for achieving a target spectral efficiency as in [13] where additive white Gaussian noise channels based on radio communications were studied.

In recent works [16, 14], the issue of delay is further investigated. Intuitively, the more hops a data packet takes to go from the source to the destination, the longer end-to-end delay it will experience. The key point is that delay is tightly coupled with reliability. In [16], the author examined the achievable error exponents when each relay node adopts simple decode-and-forward protocol, under orthogonal time-division hop scheduling. It is observed therein that increasing the number of hops eventually degrades the end-to-end reliability function. In [14], two different multihopping strategies, namely, the concatenated coding strategy and the pass-or-decode strategy, were analyzed and compared. It is observed therein that the choice of transmission strategy has a significant impact upon the delay-reliability tradeoff, and should be optimized according to the channel parameters, as well as the target information rate.

In this paper, we employ error-exponents as in [14, 16] to analyze multihop underwater acoustic networks. In contrast to wireline or wireless radio multihop networks, the propagation delay for underwater acoustic channels is significant (on the order of seconds); this is due to the slow propagation of sound in water [1]. Thus, for typical network parameters, the propagation delay for each hop is substantially larger than the allowable coding delay. In other words, because the end-to-end delay is dominated by the large propagation delay, it becomes largely immaterial for the system designer to reduce the coding delay. Coded packets are designed according to the maximally allowed block length, mainly determined by other considerations like computational complexity and hardware/software resource limitation. This characteristic is unique to underwater acoustic channels and is not present for electro-magnetic wave channels like copper, fiber, or radio.

For underwater acoustic networks, the issue of inter-hop interference is much less severe as in wireless environments. This is because of the rapid signal attenuation with path distance. As our analysis will show, for typical network parameters, the total interference from all the interfering relay nodes is tens of dBs below the desired signal power, and thus can be safely ignored for practical purposes. Furthermore, the effective transmission bandwidth is range-dependent. As such, shorter hops have larger transmission bandwidths. This is another feature unique to underwater channels and is a further impetus for the consideration of multihop networks. On the other hand, the issue of intra-hop interference, namely the intersymbol interference (ISI) due to multipath propagation, is rather significant and needs to be addressed carefully. As a simple, but practical solution, we introduce guard intervals to eliminate the effect of ISI between adjacent packets. The introduction of guard intervals impacts our overall achievable rate; however, as will be seen, even with this conservative ISI combatting method, multihopping achieves significant gains over single hop communications with respect to both reliability and rate.

In this paper, we analyze a shallow-water multihop network and consider typical network parameters. The main finding is that, increasing the number of hops within practically reasonable values is always beneficial in terms of end-to-end achievable rate and reliability. Such a conclusion is spiritually consistent with the earlier result that multihopping significantly reduces the detection symbol error rate [1]. It is also important to note that, these results only imply that purely physical-layer considerations may not yield performance tradeoff for multihop underwater acoustic networks. In fact, higher-layer analysis does exhibit interesting tradeoff between the number of hops and attainable end-to-end performance; see, *e.g.*, [18].

[1]An exception in wireless systems is deep-space communication, in which the large propagation delay stems from the extremely long path distance from the outer space to the earth [17].

The remainder of this paper is organized as follows. In Section 2, we introduce the useful background knowledge for subsequent sections. Specifically, we briefly outline the signal attenuation and noise characterizations, the delay-reliability tradeoff quantified by reliability functions, and the guard interval approach for eliminating ISI. In Section 3, we describe the multihop transmission protocol, presenting the analytical results for quantifying the end-to-end delay-reliability tradeoff. In Section 4, we provide numerical results which underscore our analysis, and make additional observations regarding the system behavior. Finally, in Section 5 we conclude the paper.

2. PRELIMINARIES

2.1 Signal Characterization for Underwater Acoustic Channels

Throughout this paper, we consider a shallow-water acoustic propagation environment, and for simplicity assume that it is spatially and temporally homogeneous. We further ignore the issues of channel variations. The following description of the underwater acoustic channel characterization is from [22]. Sound propagates through water at approximately $c = 1500$ m/s. The attenuation, or path loss that occurs over a distance l, for a narrow-band signal of carrier frequency f, is given by

$$A(l, f) = A_0 l^k a(f)^l, \quad (1)$$

where A_0 is a normalizing constant, k is the spreading factor, taken as the practical spreading $k = 1.5$, and $a(f)$ is the absorption coefficient, modeled following the Thorp's formula as [20]

$$10 \log a(f) = \\ 0.11 \frac{f^2}{1+f^2} + 44 \frac{f^2}{4100+f^2} + 0.000275 f^2 + 0.003, \quad (2)$$

in dB/km, with f in kHz.

Assuming the absence of site-specific noise, the receiver is affected by colored ambient noise only, with its overall power spectral density in units of dB re μ Pa (*i.e.*, in decibels relative to a micro Pascal) in kHz

$$N(f) = N_t(f) + N_s(f) + N_w(f) + N_{th}(f), \quad (3)$$

where [21]

$$\text{turbulence}: 10 \log N_t(f) = 17 - 30 \log f \quad (4)$$

$$\text{shipping}: 10 \log N_s(f) \\ = 40 + 20(s - 0.5) + 26 \log f - 60 \log(f + 0.03) \quad (5)$$

$$\text{waves}: 10 \log N_w(f) \\ = 50 + 7.5\sqrt{w} + 20 \log f - 40 \log(f + 0.4) \quad (6)$$

$$\text{thermal}: 10 \log N_{th}(f) = -15 + 20 \log f. \quad (7)$$

In this paper, for simplicity, we take the shipping activity factor to be $s = 0.5$ and the wind speed to be $w = 0$.

Jointly affected by the attenuation $A(l, f)$ and the noise power spectrum density $N(f)$, the nominal signal-to-noise ratio (SNR) characteristic of a narrow-band signal with unit transmit power, carrier frequency f, received at distance l

is

$$\rho(l,f) = \frac{1}{A(l,f)N(f)}. \quad (8)$$

The fact that SNR depends upon both distance and frequency has a fundamental impact upon underwater acoustic communication system design. For a given distance l, there is an optimum frequency f_l^* at which the SNR $\rho(l,f)$ is maximized as ρ_l^*. Around f_l^* we can define the 3-dB frequency range $[f_l^a, f_l^b]$ such that $\rho(l, f_l^a) = \rho(l, f_l^b) = \rho_l^* - 3$ in dB [22]. We note that, in this paper, SNR is usually represented in units of dBs for notational convenience, but should be taken as its linear-scale value in actual calculations.

2.2 Delay-Reliability Tradeoff for Channel Coding

Physical-layer information theory asserts that, for any coded transmission, there exists an inherent delay-reliability tradeoff, as quantitatively captured by the reliability function [15]. In order to transmit information reliably, the information bits must be appropriately encoded by incurring a certain amount of delay. Intuitively, for transmitting at a fixed rate, codewords with longer block length imply lower decoding error probability. Roughly speaking, the reliability function thus is defined as the decay rate of the decoding error probability as the coding block length increases asymptotically. Different types of upper and lower bounds to the reliability function have been derived; see, *e.g.*, [15]. In this paper, for simplicity, we illustrate the application of the so-called random-coding error exponent only. The random-coding error exponent is the simplest lower bound to the reliability function, and in fact is tight for rates beyond the so-called critical rate. We note that, although the concepts of error exponent and reliability function are theoretical, it has been recently found that the exponential decaying behavior of decoding error probability indeed exists for iterative decoding algorithms of certain turbo and low-density-parity-check (LDPC) codes [19].

For underwater acoustic channels with frequency-dependent channel response as characterized in Section 2.1, however, exact evaluation of those bounds is analytically complicated. In this paper, for illustrative purposes, we consider a simplified scenario to obtain lower bounds to the reliability function as well as the system performance. That is, we replace the true channel SNR characteristic $\rho(l,f)$ by an on-off function

$$\tilde{\rho}(l,f) = \begin{cases} \rho_{l,0} := \rho_l^* - 3 \text{ (dB)}, & f \in [f_l^a, f_l^b] \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

From an operational perspective, $\tilde{\rho}(l,f)$ corresponds to the idealized scenario that the transmitted signal occupies only the 3-dB frequency range $[f_l^a, f_l^b]$, and that the receive SNR over the whole 3-dB frequency range is taken as the lower bound $\rho_{l,0}$. Thus the results obtained in this paper can also be interpreted as achievable lower bounds to the optimal performance.

Using the capacity formula for additive white Gaussian noise (AWGN) channels, the capacity of the channel with the simplified SNR characteristic $\tilde{\rho}(l,f)$ is

$$\tilde{C}_l = W_l \log\left(1 + \frac{P_T \tilde{\rho}_0}{W_l}\right), \quad (10)$$

where P_T is the transmitted power, and $W_l := f_l^b - f_l^a$ is called the 3-dB bandwidth of channel. We note that $\tilde{C}_l$ is only a lower bound to the channel capacity [22], as we ignore the issues of frequency-dependent SNR and use a simplified SNR model. As in [22] we also assume a time-invariant channel.

The random-coding error exponent $E(R)$ asserts that, for sufficiently long coding length T, there exist channel codes with rate R and corresponding decoding algorithms, such that the average probability of decoding error, P_e, satisfies the exponential behavior

$$\lim_{T\to\infty} \frac{-\log P_e}{T} \geq E(R). \quad (11)$$

Following [15, Sec. 7.4], for the simplified underwater acoustic channel with $\tilde{\rho}(l,f)$, we have the following parametric form of its random-coding error exponent. Define $\text{snr}_l := P_T\rho_{l,0}/W_l$, which is the SNR in dB re μ Pa per Hz. If rate R satisfies

$$W_l \log\left\{\frac{1}{2}\left[1 + \frac{\text{snr}_l}{2} + \sqrt{1 + \frac{\text{snr}_l^2}{4}}\right]\right\} \leq R \leq \tilde{C}_l, \quad (12)$$

we have

$$\begin{aligned} R &= W_l \log\left(\beta + \frac{\text{snr}_l}{1+\alpha}\right), \quad (13)\\ E_l(R) &= W_l\left[\log\beta + (1+\alpha)(1-\beta)\right], \quad (14)\end{aligned}$$

where for each $0 \leq \alpha \leq 1$, β is given by

$$\beta = \frac{1}{2}\left\{1 - \frac{\text{snr}_l}{1+\alpha} + \sqrt{(1 - \frac{\text{snr}_l}{1+\alpha})^2 + \frac{4\text{snr}_l}{(1+\alpha)^2}}\right\}. \quad (15)$$

If rate R satisfies

$$0 \leq R < W_l \log\left\{\frac{1}{2}\left[1 + \frac{\text{snr}_l}{2} + \sqrt{1 + \frac{\text{snr}_l^2}{4}}\right]\right\}, \quad (16)$$

we have

$$\begin{aligned} E_l(R) &= W_l \log\left\{\frac{1}{2}\left[1 + \sqrt{1 + \frac{\text{snr}_l^2}{4}}\right]\right\} + \\ & \quad W_l\left[1 + \frac{\text{snr}_l}{2} - \sqrt{1 + \frac{\text{snr}_l^2}{4}}\right] - R. \quad (17)\end{aligned}$$

2.3 Shallow-Water Multipath Model

A full account of the multipath propagation should involve all possible paths that the transmitted wave travels toward the receiver. In this paper, for simplicity, we assume that all the nodes are at the same depth in a shallow-water environment. Therefore, besides the direct path, we only consider the dominant interfering path that is reflected by the bottom. This situation is illustrated in Figure 1. From simple geometric relationship, the arrival of the interfering signal is delayed by

$$\epsilon = \frac{1}{c} \cdot \left[\sqrt{r^2 + (2h)^2} - r\right] \quad (18)$$

in seconds. If the bottom depth h is substantially smaller than the inter-node distance r, then we can apply the following approximation:

$$\epsilon \approx \frac{2h^2}{rc}. \quad (19)$$

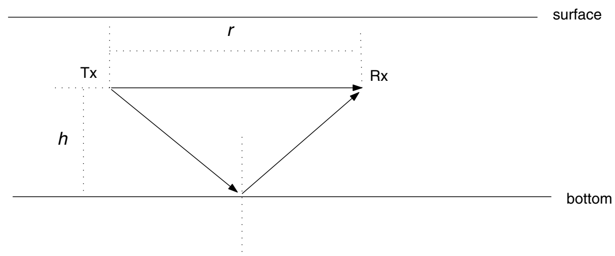

Figure 1: Illustration of the shallow-water multipath model.

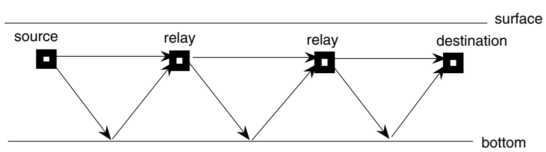

Figure 2: Illustration of the multihop network.

For typical network configurations, the nodes distance is of the order of several kilometers, while the shallow water has depth tens of meters. In that case, such an approximation is roughly satisfied.

Our objective in establishing this simple model is to assess the effects of inter-symbol interference as we increase the number of hops. With our model, it is clear to see that the propagation delay of the ground bounce becomes relatively larger than that of the direct path as we increase the numbers of hops and thus, reduce the inter-node distance. Thus, to achieve inter-symbol interference avoidance, we can insert guard intervals of an appropriate time; however, this will affect our overall achievable rate. We seek to determine how this increase in guard time with an increase in hops will affect the relationship between reliability and rate.

3. MULTIHOP TRANSMISSION PROTOCOLS

In the multihop transmission problem considered in this paper, the source and the destination are fixed and d km apart, and $(K-1)$ relay nodes are uniformly located between the source and the destination, such that the resulting K hops have identical path distances and homogeneous channel characteristics. From Section 2.1, these hops can be parametrized by the common hop distance, d/K km. All the nodes are located near the water surface and have the same depth. Figure 2 illustrates the network configuration.

An important property of the current implementation of underwater acoustic modems is that they are half-duplex, *i.e.*, a modem cannot transmit and receive simultaneously. Therefore some form of hop scheduling is required to transmit information from the source to the destination hop by hop. In this paper, we focus on the case that all the relay nodes follow decode-and-forward scheme. Each packet is a coded block with a fixed time duration τ_c s, and is decoded and re-encoded at each relay node. The value of τ_c is determined by higher-layer protocols, and is also constrained by implementation complexity, therefore is typically a number substantially smaller than the propagation delay.

The total end-to-end propagation delay is a constant d/c, and each hop has a propagation delay

$$\tau_p = \frac{d}{Kc}. \tag{20}$$

From the shallow-water multipath model in Section 2.3, the arrival delay of the bottom reflected path for each hop is

$$\epsilon_K \;=\; \frac{1}{c}\cdot\left[\sqrt{(\frac{d}{K})^2+4h^2}-\frac{d}{K}\right], \tag{21}$$

where h is the bottom depth. In this paper we assume that $K \ll d/h$, then it is easily seen that $\epsilon_K \ll \tau_p$, *i.e.*, the propagation delay dominates the total delay.

For a given number of hops K, the hop propagation delay τ_p is fixed, whereas we have a certain flexibility in choosing the guard interval for eliminating the ISI as well as efficiently scheduling hop access. A particular case occurs when the ratio

$$\frac{\tau_p}{\tau_c+\epsilon_K} = q, \tag{22}$$

for some odd integer $q = 1, 3, \ldots$. If this is the case, then we can schedule the $(K+1)$ nodes (source, destination, and $(K-1)$ relay nodes) such that at any time, half (if K is odd; or almost half, if K is even) of the nodes are transmitting while the remaining nodes are receiving. So there is no “leakage” in terms of time utilization. To illustrate this, consider a simple example in which $q = 1$ and $K = 1$. Let the data packets be sent by the source node during time intervals $[2j\tau_p, (2j+1)\tau_p]$ for $j = 0, 1, \ldots$, then they arrive at the relay node during time intervals $[(2j+1)\tau_p, (2j+2)\tau_p]$. Therefore the relay node can schedule its transmission during time intervals $[2j\tau_p, (2j+1)\tau_p]$, without any conflict between transmission and reception. The resulting overall rate is reduced by a factor of two. In general, the ratio between τ_p and $(\tau_c + \epsilon_K)$ is not exactly an odd integer and a certain degree of timing imperfection always exists. However, because for typical network parameters we have $\tau_c, \epsilon_K \ll \tau_p$, we can always slightly adjust the coding block duration τ_c such that the ratio is modified to an odd integer.

3.1 Effect of Inter-Hop Interference

Because there exist concurrent transmissions in the hop sche-duling scheme, the effect of inter-hop interference, *i.e.*, the interference caused by undesired transmitting nodes to a receiver, needs to be taken into account. Unlike radio channels, underwater acoustic channels tend to exhibit a rather desirable interference mitigation property. The key underlying reason for this is that the SNR characteristic $\rho(l, f)$ depends upon distance as well as frequency. For two different values of distance l, $\rho(l, f)$ is maximized at different frequencies f. For a receiving node, since the interferer nodes are located at least three times the distance of the desired transmitting node, the interference power level within the 3-dB frequency range of the desired signal is usually more than ten dBs lower than the signal power level, for typical network parameters. Based on this analysis, we see that the inter-hop interference can be ignored as it is negligible. To

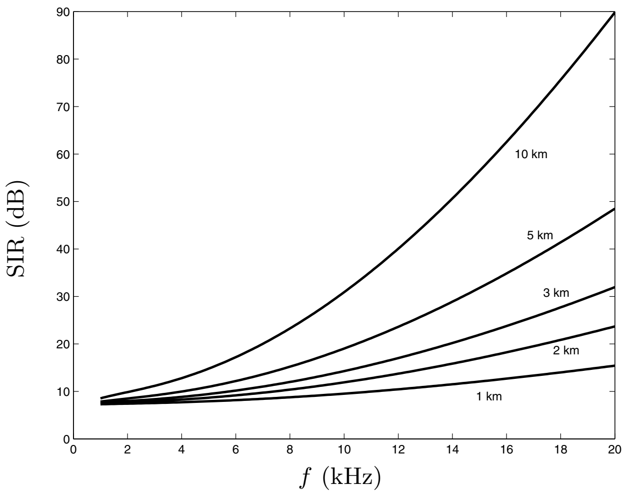

Figure 3: The signal-to-interference ratio (SIR) $10\log\left[\frac{\rho(l,f)}{\rho(3l,f)}\right]$ for different link distances.

see this quantitatively, note that the SNR characteristics for two distances, l and l', at frequency f, is

$$\begin{aligned} & 10\log\left[\frac{\rho(l,f)}{\rho(l',f)}\right] \\ = & \ 10\log\left[\frac{A(l',f)}{A(l,f)}\right] \\ = & \ k\cdot 10\log(l'/l)+(l'-l)\cdot 10\log a(f). \end{aligned} \quad (23)$$

Consider the case that $l' = 3l$, then (23) is the signal-to-interference ratio (SIR) caused by the nearest (*i.e.*, dominant) interfering node for a desired link with distance l. Using the practical spreading factor $k = 1.5$, and Thorp's formula (2), we have

$$\begin{aligned} & 10\log\left[\frac{\rho(l,f)}{\rho(3l,f)}\right] \\ = & \ 7.1568+\Big[0.22\cdot\frac{f^2}{1+f^2}+88\cdot\frac{f^2}{4100+f^2} \\ & +0.00055\cdot f^2+0.006\Big]\cdot l \end{aligned} \quad (24)$$

in dB, with l in km, as plotted in Figure 3 for different typical values of l. From Figure 3 we can notice that the SIR is fairly large, and rapidly grows with frequency, which is a desirable property for shorter distances because then the 3-dB bandwidth is also shifted to higher frequencies. Therefore for practical purposes we can safely ignore the effect of inter-hop interference, or in the worst case simply introduce a small SNR offset.

3.2 Interplay among Rate, Delay, and Number of Hops

Due to hop scheduling and guard intervals, if we target achieving an end-to-end information rate of R bits/s, the channel coding for each hop needs to achieve rate $R' := 2R(1+\epsilon_K/\tau_c)$ bits/s. On the other hand, by tracking the transmission of one packet, *i.e.*, codeword, we find that the

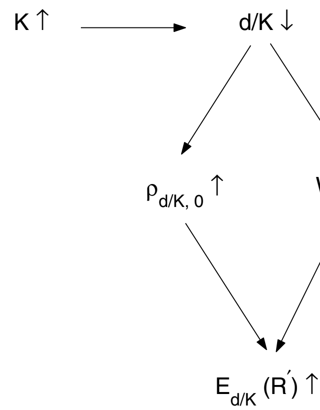

Figure 4: Illustration of the gain for end-to-end reliability by increasing the number of hops K.

end-to-end delay for transmitting a packet is

$$\begin{aligned} \bar{\tau} &= K\cdot(\tau_p+\epsilon_K+\tau_c) \\ &\approx K\cdot\tau_p \\ &= \frac{d}{c}. \end{aligned} \quad (25)$$

Here we have utilized the fact that $\tau_c, \epsilon_K \ll \tau_p$. So we notice that the end-to-end delay $\bar{\tau}$ is essentially dominated by the end-to-end propagation delay d/c, regardless of the number of hops K, as long as the conditions we have assumed are satisfied.

As the end-to-end delay $\bar{\tau} = d/c$ is fixed, we can invoke the delay-reliability tradeoff in Section 2.2 to lower bound the achievable end-to-end reliability function. The union bounding argument asserts that, if the end-to-end probability of decoding error is P_e, then the allowed probability of decoding error for each hop should be no greater than P_e/K. Since for practical systems P_e is typically close to zero, both P_e and P_e/K exhibit the same asymptotic decay rate. Specifically, for small error rates the end-to-end probability of error P_e is well approximately by

$$\begin{aligned} \frac{-\log P_e}{\bar{\tau}} &\approx \frac{-\log(P_e/K)}{\tau_c}\cdot\frac{\tau_c}{\bar{\tau}} \\ &\geq \tilde{E}_K(R) := E_{d/K}(R')\cdot\frac{c\tau_c}{d}. \end{aligned} \quad (26)$$

In view of $\tilde{E}_K(R)$, we notice that increasing K boosts the end-to-end reliability, as the chart in Figure 4 illustrates.

Thus, there are two benefits by increasing the number of hops, or, equivalently reducing the hop distance, for underwater acoustic networks. First, it leads to an increase in the hop SNR, which is far more significant than that for radio wireless channels, because the attenuation contains an exponential term with the hop distance. Second, and also quite unique not present in radio wireless channels, it also leads to an increase in the usable hop bandwidth, because of the frequency-dependent attenuation characteristic. The two effects thus results to a boosted increase for the end-to-end reliability. However, our use of guard-bands to avoid inter-symbol interference may result in a tradeoff with these effects. As noted before, as the number of hops increase, the number of necessary guard bands and the relative duration of the guard bands increase. In fact, as the numerical results

Table 1: Key hop parameters for the numerical example in Section 4

K	d/K (km)	$f^*_{d/K}$ (kHz)	$\rho^*_{d/K}$ (dB)	$W_{d/K}$ (kHz)	$\rho_{d/K,0}$ (dB)
1	9	6.25	−97.2	8.1	−100.2
2	4.5	9.15	−89.43	11.5	−92.43
3	3	11.35	−84.96	14.25	−87.96
4	2.25	13.25	−81.81	16.64	−84.81
5	1.8	14.9	−79.37	18.73	−82.37
6	1.5	16.4	−77.39	20.62	−80.39
7	1.29	17.8	−75.71	22.25	−78.71
8	1.13	19.05	−74.27	23.76	−77.27
9	1	20.15	−73.01	25.13	−76.01
10	0.9	21.2	−71.88	26.32	−74.88

will show, the performance gains of multihopping dominate over the effects that deteriorate performance.

We finally remark that, the validity of the above analysis only holds when the number of hops, K, is not too large. More precisely, the following conditions need to be satisfied:

$$K \ll \min\left\{\frac{d}{c\tau_c}, \frac{d}{h}\right\}. \tag{27}$$

That is, the total coding delay along the K hops should be substantially smaller than the end-to-end propagation delay, and the bottom depth should be substantially smaller than the hop distance.

4. NUMERICAL RESULTS

In this section, we illustrate the analytical results in the previous sections through a detailed study of a numerical example. Specifically, we fix the source-destination distance $d = 9$ km, and investigate how the number of hops affects the system behavior. The node-bottom distance is taken as $h = 90$ m. Therefore the hop propagation delay is $\tau_p = 9 \times 10^3/(1500 \times K) = 6/K$ s, and the arrival delay (also the guard interval duration) is

$$\begin{aligned}\epsilon_K &= \frac{1}{1500}\left[\sqrt{(\frac{9000}{K})^2 + 4 \times 90^2} - \frac{9000}{K}\right] \\ &= 0.06 \times \left[\sqrt{\frac{10000}{K^2} + 4} - \frac{100}{K}\right],\end{aligned} \tag{28}$$

is in seconds. It is evident that $\epsilon_K \ll \tau_p$ for values of K below ten. For example, when $K = 10$, we still have $\tau_p/\epsilon_K > 50$. The coded packet duration is chosen as $\tau_c = 0.05$ s, which roughly corresponds to 1000 channel degrees of freedom if the bandwidth is 10 kHz. Again, we note that $\tau_c \ll \tau_p$ for a wide range of K. The scaled rate R' is

$$\begin{aligned}R' &= 2R \cdot \left(1 + \frac{\epsilon_K}{\tau_c}\right) \\ &= \left[2.4 \times \sqrt{\frac{10000}{K^2} + 4} - \frac{240}{K} + 2\right] \times R.\end{aligned} \tag{29}$$

We then determine the channel characteristics of the K hops. The key parameters include the hop distance d/K, the optimum frequency $f^*_{d/K}$ and the maximized SNR $\rho^*_{d/K}$ (see Section 2.1), the corresponding 3-dB bandwidth $W_{d/K}$ and the discounted SNR $\rho_{d/K,0}$ (see Section 2.2). For K from 1 to 10, we calculate these parameters and summarize them in Table 1.

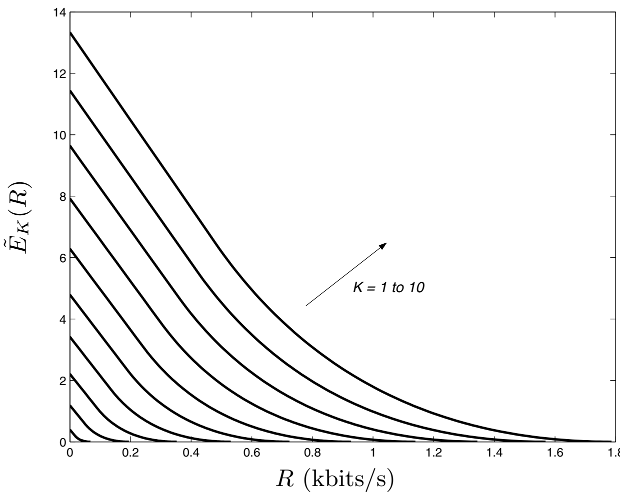

Figure 5: The achievable end-to-end reliability function for the numerical example.

From Table 1 we observe that, although the hop SNR and bandwidth both increase with K, their growth rates slow down as K gets large. Therefore we expect that the benefits of multihopping will become marginal for sufficiently large K. However, for the values of K examined in this paper, increasing K always leads to significant performance gains.

For $K = 1$ to 10, we numerically compute the achievable reliability function

$$\begin{aligned}\tilde{E}_K(R) &= E_{d/K}(R') \cdot \frac{c\tau_c}{d} \\ &= E_{d/K}(R') \cdot \frac{1500 \times 0.05}{9 \times 10^3} \\ &= 8.33 \times 10^{-3} \times E_{d/K}(R').\end{aligned} \tag{30}$$

The total transmit power is 120 dB re μ Pa, and is equally allocated among the source and the $(K-1)$ relay nodes. Figure 5 displays the numerical curves of $\tilde{E}_K(R)$.

From Figure 5, we observe that the increases in both end-to-end rate and end-to-end reliability with K are evident. Comparing the single-hop case ($K = 1$) and the 10-hop case, we obtain more than ten-fold performance boost. It is also interesting to notice the rule of thumb that all the curves are approximately parallel.

5. CONCLUSIONS

In this paper, we provide a preliminary analysis of a delay-reliability tradeoff for multihop underwater acoustic networks. Unlike for other propagation media such as copper, fiber, and radio, a key property of underwater acoustic channels is that the propagation delay is typically the dominant component of the total end-to-end delay, due to the extremely slow speed of sound in water. Therefore instead of exhibiting a tradeoff between the number of hops and end-to-end performance (as in our earlier work [14]), multihop underwater acoustic networks always favor more hops, for typical network parameters. Our numerical results illustrate that, multiple folds of performance boosts are theoretically realizable from multihopping. Such observations are also consistent with earlier results regarding bit error probability only [1].

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