Poster Abstract: An Underwater Positioning Scheme for 3D Acoustic Sensor Networks

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Abstract—We transform the 3D underwater positioning problem into its 2D counterpart via a projection technique that employs depth information available to underwater sensors. After showing that a non-degenerative projection preserves network localizability, we present a purely distributed localization scheme termed USP for underwater acoustic sensor networks. In USP, reference nodes are projected to the plane that contains a to-be-localized sensor, and elegant two dimensional localization techniques are applied when the projection is non-degenerative. USP is shown to have improved localization capabilities over existing 3D methods through theoretical analysis and extensive simulation.

I. INTRODUCTION

Underwater sensor networks (USNs) consist of a variable number of sensors designed to collaboratively monitor an oceanic environment. Despite the attractiveness of location information to many USN applications, location discovery is a non-trivial problem in the underwater medium where these networks are deployed. Indeed, propagation delays, motion-induced Doppler shift, and multipath interference render many localization techniques inaccurate or infeasible (e.g., schemes based on TDoA or RSSI) [1]. Even the well-established Global Positioning System (GPS) does not work well underwater [2]. This raises the question of how to design a simple distributed algorithm that computes a position for all localizable nodes in the networks under study.

Our idea is to non-degeneratively project the locations of references nodes (anchor nodes or previously localized nodes) to the plane containing a to-be-localized node. We have the following two contributions. We first prove that each node preserves its localizability in the plane that it is projected on if the projection is non-degenerative. Specifically, a node is uniquely (finitely) localizable in its projection plane if and only if it is uniquely (finitely) localizable in the original 3D network. We then design and extensively analyze a purely distributed underwater sensor positioning scheme termed USP that employs our provably effective projection technique.

II. NETWORK LOCALIZABILITY STUDY

The network localization problem refers to determining a unique position for each node in a network given the position of some anchor nodes (or “reference nodes”) and the knowledge of some inter-node distances. Although the complexity of uniquely localizing a network is NP-hard [3], there exist classes of networks that can be efficiently localized. Examples of such classes include bilateration networks in one dimension; trilateration networks in two dimensions; and quadrilateration networks in three dimensions. Quadrilateration networks can be considered as the 3D counterpart to trilateration networks. Furthermore, in three dimensions, quadrilateration networks are the only uniquely localizable networks, while bilateration networks and trilateration networks are finitely localizable.

However, in 3D underwater acoustic sensor networks, we can employ a sensor’s depth information to map the positions of reference nodes to locations in the projection plane where a to-be-localized underwater sensor resides. This mapping effectively transforms the problem of 3D underwater localization into a 2D positioning problem such that many of the elegant localization techniques for 2D networks become applicable. Our projection technique is illustrated in Fig. 1, and we formally show its effectiveness below.

\[ F: \mathbb{E} \rightarrow \mathbb{R}^3 \]

\[ F = (V, E) \]

\[ G_F = (V_F, E_F) \]

\[ P_F : \mathbb{E} \rightarrow \mathbb{E} \]

\[ P_F(u) = \begin{cases} \frac{1}{d(u)} \cdot \frac{1}{d(u)} u & \text{if } u \in \mathbb{E} \setminus \{v\} \\ v & \text{if } u = v \end{cases} \]

Definition 1: Given a plane \( F \) in three-dimensional space, a projection \( P_F : \mathbb{E} \rightarrow \mathbb{E} \) that maps a node \( u \) in 3D to a node \( u' \) in the plane \( F \), i.e., \( P_F(v) = v' \). Note that \( P_F \) is a Euclidean transformation.

Definition 2: Given a 3D graph \( G = (V, E) \) and a plane \( F \), the projection graph \( G_F = (V_F, E_F) \) is produced by the projection \( P_F \), with \( V_F = \{v' | v \in V \} \) and \( E_F = \)
\{(v_i^F, v_j^F) \mid (v_i, v_j) \in E, \ i \neq j\}.

**Theorem 1:** Assume the projection \(P_F\) is non-degenerate, i.e., no three (projected) reference nodes are collinear. Given a 3D graph \(G = (V, E)\) and a plane \(F\), where the relative distance from a node \(v\) to \(F\) is known \(\forall v \in V\), a node \(v\) is localizable in \(G\) if and only if \(v^F\) is localizable in \(G_F\).

**Corollary 1:** If \(P_F\) is bijective, then \(G = (V, E)\) is uniquely (finitely) localizable if and only if \(G_F = (V_F, E_F)\) is uniquely (finitely) localizable in the projection plane \(F\).

Note that Theorem 1 and Corollary 1 allow us to conclude that a non-degenerate projection preserves the localizability of a network \(G\). Based on this observation, we elucidate the design of our distributed underwater sensor positioning scheme termed USP in the next section.

**III. USP Design**

We consider three dimensional USNs where nodes are randomly distributed throughout an oceanic medium. Practical issues such as economics and propagation characteristics of radio waves in water dictate that sensors will be sparsely deployed [4] and use acoustic waves for communication [5], respectively. Furthermore, acoustic wireless communication suggests that ToA be used when sensors need to measure distances between themselves [1]. Also employed by USP is the depth information available to each sensor [5].

USP contrasts with traditional 3D underwater localization schemes such as silent positioning [6] in that it does not require the existence of at least 4 non-coplanar anchor nodes (or reference nodes) within communication range of a to-be-localized node. This requirement is obviated by USP through a novel combination of sensor depth information and location projection from one plane to another.

Two main phases compose USP: an offline pre-distribution phase and a distributed localization phase. The first phase consists of nodes being pre-loaded with initial configuration information (e.g., the amount of the time allocated for each iteration), while the latter is executed a maximum number of iterations \(n\) by each of the deployed nodes in a distributed manner. Three time periods denoted as \(\Delta_B, \Delta_C, \text{ and } \Delta_S\), contribute to the total time required for each iteration.

The first time period, \(\Delta_B\), has each sensor perform a local broadcast of any new position information that it has. This information is available when a node is only deployed (when a node is an anchor) or when a node’s location information is updated from a computation operation or a reduction operation performed during the previous iteration. Each sensor also records updated position information corresponding to neighbors from which it receives broadcasts.

Sensors next compute position information during time period \(\Delta_C\). If a sensor has no previous position information, it attempts to compute its position using the projection technique and the position information of its neighbors. Alternatively, if a sensor already has position information available, it attempts to reduce its set of candidate of positions.

Lastly, all sensors sleep for a period of \(\Delta_S\). After completion of this step, an iteration of total length \(\Delta_T\) has finished, and the subsequent iteration of USP begins.

**IV. Evaluation**

The performance of USP is evaluated through both theoretical analysis and extensive simulation.

**Definition 3:** Given a three-seed subgraph \(G_{ts}\) of \(G\), a **maximal bilateration extension subgraph** \(G_{m} = (V_{m}, E_{m})\) of \(G\) is a bilateration extension subgraph such that \(\forall v \in V \setminus V_{m}, |N(v) \cap V_{m}| < 2\), where \(N(v) = \{v_{i} | (v_{i}, v_{j}) \in E\}\)

Note that the set of nodes finitely localized by USP has a bilateration ordering. Specifically, when USP terminates, the set of finitely localized nodes has a maximal bilateration ordering, which induces a maximal bilateration extension graph that is itself a bilateration extension subgraph of \(G\).

**Theorem 2:** The set of nodes that can be localized with one maximum bilateration ordering can also be localized with the others.

USP can therefore localize all nodes localized by a bilateration method. Further illustration of the localization capabilities of USP appears in Fig. 2. We compare the percentage of the total number of nodes that are uniquely localized by USP to that of the traditional quadrilateration method employed in three dimensions with respect to the average node degree. Also identified is the number of nodes that USP finitely localizes. Observe that USP significantly outperforms the traditional 3D method in sparse underwater sensor networks.

![Fig. 2. The ratio of nodes localized by USP, quadrilateration, and finitely localized with respect to average node degree.](image_url)

**V. Future Work**

As part of our future work, we plan to have USP include a network partitioning and joining strategy in order to reduce the total number of iterations and the accumulated error.

**REFERENCES**


